between the integral flow parameters is less. As an example, some data for three calculation variants are given in Table 1 (the numerator shows percentage values of ζ, %, and the denominator shows values of δ_{43}^{a} in μm).

Therefore, when accounting for the effect of gas flow, the decrease in momentum loss due to refinement of Φ_{ji} is about 10% of ζ.

LITERATURE CITED

1. L. E. Sternin, B. N. Maslov, A. A. Shrayber, and A. M. Podvysotskii, Two-Phase Monodisperse and Polydisperse Gas Flow with Particles [in Russian], Mashinostroenie, Moscow (1980).
2. A. M. Podvysotskii and A. A. Shrayber, "Calculations of nonequilibrium two-phase flow with coagulation and splitting of condensation particles for arbitrary mass and velocity distributions of secondary drops," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2 (1975).
3. A. A. Shrayber, A. M. Podvysotskii, and B. N. Maslov, "The effect of gas flow on the splitting of drops in Laval nozzles," Prom. Teplotekhnika, 4, No. 4 (1982).
4. V. A. Arkhipov, V. P. Bushlanov, et al., "Equilibrium forms and stability of rotating drops," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 4 (1982).

STUDY OF INERTIAL SETTLING OF POLYDISPERSED PARTICLES AT THE CRITICAL POINT OF A SPHERE

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The flow of an incompressible gas with particles past a body at high Reynolds numbers is studied in many works, for example, in [1-6], where in the calculation of the characteristics of inertial settling of an impurity the particles are assumed to be monodispersed. At the same time, in real gas suspensions the particle sizes are always different. Polydispersity of particles even in the case when their interaction with one another is ignored, substantially complicates the picture of the motion of the impurity near the body. Particles of different sizes are deflected by the gas flow differently. As a result, the fractions are redistributed in space and the initial particle-size distribution function of the average density of the dispersed phase changes. In this case it is difficult to set up and solve the "kinetic" equation describing the evolution of the distribution function. In this paper we propose a method for calculating the flux density of settling polydispersed particles at the front critical point and the flux-density distribution function over the fractions. In so doing just as in [1-6], it is assumed that the particle concentration is small, and the effect of the particles on the gas flow and the interaction of particles with one another are ignored.

In the case when the impurity concentration is negligibly small, the problem of the flow of a gas suspension past a body reduces, as is well known, to a sequence of two simpler problems the construction of the flow field of the carrying medium near the body and the calculation of the particle trajectories in this field. If the Reynolds number is large, then the viscosity of the gas in the problem of flow past the body is usually ignored. Estimates [1, 3, 4] and a direct calculation [7] show, however, that there exists a quite wide range of parameters of the flow of the gas suspension where the viscous boundary layer on the surface of the body substantially affects the motion of the impurity and, therefore, in the general case it cannot be neglected in determining the characteristics of inertial settling of the particles. In this paper the flow field of the gas near the sphere is given just as in [7], based on a model which includes the external potential flow and the viscous boundary layer. It is shown in [8] that the use of such a model in the calculation of the flux density of the settling particles gives a quite high accuracy at the critical point, if Re $\gtrsim 10^5$.

In the problem under study the dominant force exerted by the carrying gas on a dispersed particle is the aerodynamic drag force [1, 4, 7]. Stokes' law [1-4] or the "standard curve" [5-7], which is obtained for an unbounded uniform gas flow past a particle, is often used for the aerodynamic drag coefficient of the particle. At the same time, it is known [9] that when the particle motion in the viscous medium is slow, near a solid surface its aerodynamic

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drag coefficient can be several times higher than the value obtained from Stokes' formula. Because of this, in this paper we evaluate the effect of the wall on the trajectory of the particles near the critical point.

The numerical results presented refer to the log-normal distribution law for the average particle density over size fractions in the unperturbed flow. The characteristics of inertial settling of the dispersed phase as a function of the Reynolds number and the parameters r_m and s, entering into the log-normal law, are studied.

1. Let a uniform flow of a gas suspension be incident with a velocity V_∞ on a sphere radius a. The carrier gas is assumed to be viscous and incompressible, and the particles are assumed to be spherical. We shall study a neighborhood around the axis of symmetry in front of this sphere, where the flow is laminar. It is assumed that the effect of the particles on the motion of the gas is negligibly small and the particles do not interact with one another. The equations of motion of a two-phase mixture [10] decompose in this case into the equations of motion of the pure gas

$$\operatorname{div} \mathbf{V} = 0, \quad \rho^0 \frac{d\mathbf{V}}{dt} = \operatorname{div}(-pE + \tau) \tag{1.1}$$

and the equations of motion of separate particles

$$\frac{4}{3}\pi r^3 \rho_p^0 \frac{d_p \mathbf{V}_p}{dt} = \frac{1}{2} C_D \rho^0 \pi r^2 |\mathbf{V} - \mathbf{V}_p| (\mathbf{V} - \mathbf{V}_p). \tag{1.2}$$

Here $\mathbf{V}$ and $\mathbf{V}_p$ are the velocity of the gas and the velocity of the particle; ρ^0 and ρ_p^0 are the physical densities of the gas and of the particle; p is the pressure in the gas; E is the unit tensor; τ is the Navier–Stokes viscous stress tensor; r and C_D are the radius and aerodynamic drag coefficient of the particle; d/dt and d_p/dt are the substantial derivatives, associated with the motion of the gas and of the particles, respectively.

In Eq. (1.2) only the aerodynamic drag force of the particle is taken into account. It is evident from this equation that the velocity field of the carrying medium $\mathbf{V}$ must be known in order to determine the motion of separate particles. In the general case the problem of determining the field $\mathbf{V}$ requires numerical integration of the full Navier–Stokes equations (1.1).

We shall study a state of flow past a sphere when $\mathrm{Re} = V_\infty a/\nu$ is large (ν is the kinematic viscosity of the gas). In this case, in order to solve the problem of flow of a pure gas over the bow surface of the sphere, the method of exterior and interior asymptotic expansions with respect to the small parameter $\delta = 1/\sqrt{\mathrm{Re}}$ [11] can be used; for the leading terms of the expansions it leads to Euler's equations in the exterior region and to the usual boundary-layer theory in the interior region. We shall confine our attention below to the leading terms of the expansions in each of the regions.

We introduce the boundary-layer system of coordinates (x, y), where the x axis is oriented along the generatrix of the sphere and the origin is located at the front critical point, while the y axis is oriented along the normal to x and is measured from the surface. We transform to dimensionless variables; all linear dimensions are sealed to the radius of the sphere a; the components of the gas velocity u and v and of the particle velocity u_p and v_p are sealed to v_∞; and, the time t is sealed to a/V_∞. We denote by ρ the ratio of the densities ρ^0/ρ_p^0. Then Eq. (1.2) in the boundary-layer coordinate system is written in the form

$$\frac{d_p u_p}{dt} = -\frac{u_p v_p}{1+y} + \frac{3\rho}{8r} C_D |\mathbf{V} - \mathbf{V}_p| (u - u_p),$$
$$\frac{d_p v_p}{dt} = \frac{u_p^2}{1+y} + \frac{3\rho}{8r} C_D |\mathbf{V} - \mathbf{V}_p| (v - v_p), \quad |\mathbf{V} - \mathbf{V}_p| = [(u - u_p)^2 + (v - v_p)^2]^{1/2}. \tag{1.3}$$

The components of the velocity of the carrying gas in the exterior region are calculated from the formulas of a potential flow past the sphere [12]:

$$u = [1 + (1/2)(1 + y)^{-3}] \sin x, \; v = -[1 - (1 + y)^{-3}] \cos x. \tag{1.4}$$

In the viscous boundary layer on the sphere we have [13]

$$u = (3/2) f'(\eta) x + O(x^3), \quad v = -\sqrt{3}\delta f(\eta) + O(x^2), \quad \eta = \sqrt{3} y/\delta, \tag{1.5}$$

where the function f(η) is determined from the solution of the boundary-value problem

$$2f''' + 2ff'' - f'^2 + 1 = 0, \; f(0) = f'(0) = 0, \; f'(\infty) = 1.$$

The fields of the components u and v near the sphere are obtained "in the whole" by joining the like profiles (1.4) and (1.5) at their points of intersection for each fixed value of x, as was done in [7]. The approximation of the field u constructed in this manner has a uniform error of the order $O(\delta)$ in the entire region of flow studied; for the v field this error is nonuniform. It is of the order of $O(\delta^2)$ in the boundary layer and increases up to $O(\delta)$ in the exterior nonviscous region. A more accurate approximation of the v field in the viscous layer, where $v \sim \delta$, enables a better description of the qualitative features of the behavior of the v component and has a much lower relative error than the everywhere uniform approximation, of the order of $O(\delta)$. The error in the approximation of the u and v fields gives rise to some error in the calculation of the flux density of the settling particles in separate fractions at the bow point of the sphere and in the determination of the critical radius of the particles r_* at which the smaller fractions practically do not settle on the sphere. With Re = 10^5 the error is equal to 1.5% for r_* and 1-3% for the flux density when $r > r_*$ (excluding fractions with sizes near r_*); in addition, this error decreases rapidly as Re increases [8].

To calculate the trajectory of a separate particle, the kinematic dependences

$$d_p x/dt = u_p/(1+y),\ d_p y/dt = v_p, \tag{1.6}$$

must be studied together with Eqs. (1.3) and the relations for the aerodynamic drag coefficient of a particle and the initial conditions must be given.

In the unperturbed flow, in the limit $y \to \infty$ the velocity of the particles is determined by the relations $u_p = \sin x$, $v_p = -\cos x$. In the numerical integration of the system of equations (1.3) and (1.6), however, these conditions must be given with some finite value of y_∞. Let z_∞ be the distance between the particle and the axis of symmetry at y_∞. Then the initial conditions for the starting system of equations can be written in the form

$$t=0:\ u_p = z_\infty/(1+y_\infty),\quad v_p = -\left(1-u_p^2\right)^{1/2},$$
$$x = \arcsin u_p,\ y = y_\infty.$$

In the calculations it was assumed that y = 4. In this case the components u and v of the gas velocity differ from the unperturbed values by less than 1%.

The system of equations (1.3) and (1.6) was integrated by a numerical method of the predicator–corrector type with fourth-order accuracy. First, the intermediate values of the functions sought were calculated using a four-point Adams–Bashfort difference scheme, after which these values were refined using the residual terms of the predicator at the preceding step (this procedure is described in detail in [14]); then a recalculation was performed using the four-point Adams–Multon difference scheme. In order to increase the accuracy of the calculation in the region of the viscous boundary layer, where the gradients of the flow parameters are large, the integration step here was decreased by a factor of 4-10.

The criterion for settling of a particle in the r fraction on the surface of the sphere was taken as the condition that the center of the particle intersect the line y = r. In all calculations the ratio of the phase densities p was assumed to be equal to 0.0005.

2. The dependence for C_D, approximating the "standard curve" of the drag of the sphere to within 1-2%, is proposed in [5] and has the form

$$\begin{array}{llll}
C_D = A + B/\mathrm{Re}_p + C/\mathrm{Re}_p^2, & \mathrm{Re}_p = 2|\mathbf{V}-\mathbf{V}_p|r/\nu, & & \\
\mathrm{Re}_p \leqslant 0.1, & A = 0, & B = 24, & C = 0, \\
0.1 < \mathrm{Re}_p \leqslant 1, & A = 3.69, & B = 22.73, & C = 0.0903, \\
1 < \mathrm{Re}_p \leqslant 10, & A = 1.222, & B = 29.1667, & C = -3.8889, \\
10 < \mathrm{Re}_p \leqslant 100, & A = 0.6167, & B = 46.5, & C = -116.67, \\
100 < \mathrm{Re}_p \leqslant 1000, & A = 0.3644, & B = 98.33, & C = -2778.
\end{array} \tag{2.1}$$

Under the assumptions made above, the use of the "standard curve" for the aerodynamic drag is justified if the particle moves far away from the surface in the flow. When the motion of the particle is slow, however, in the viscous boundary layer at a distance of the order of its radius from the surface of the body the drag law differs substantially from the "standard" law [9]. This is associated with the fact that the flow field of the carrying medium near the particle "at infinity" is substantially nonuniform.

A detailed review of studies of the motion of particles near solid walls based on the stationary Stokes equations is given in [9]. The degree to which the drag of the particle for $Re_p \ll 1$ differs from Stokes law depends on the relative distance between the particle and the wall, on the direction of the particle motion, and on the nature of the flow of the carrying medium near the wall. If the spherical particle moves in a viscous stationary medium near a flat unbounded wall, then for $r/h \ll 1$ the expression for C_D can be represented in the form of an approximate asymptotic formula [terms of the order of $(r/h)^3$ and higher are dropped]

$$C_D = \frac{24}{\mathrm{Re}_p}\left[1 + \beta\frac{r}{h} + \left(\beta\frac{r}{h}\right)^2\right], \tag{2.2}$$

where h is the distance between the center of the particle and the wall; the coefficient β depends on the direction of motion of the particle. Thus $\beta = 9/16$ for particle motion parallel to the wall and $\beta = 9/8$ for motion perpendicular to the wall [9]. Because of the linearity of the problem in the Stokes formulation, the motion of the particle in an arbitrary direction can be obtained as the sum of motions in the indicated directions. The formula (2.2) for parallel and perpendicular motions of the particle is compared with the corresponding exact solutions in [15]. This formula in both cases has a very high accuracy for $r/h < 0.2$. The error increases with r/h and when $r/h \gtrsim 0.65$ it is equal to about 0.5% for parallel motion and about 25% for perpendicular motion. In the limit $r/h \to 1$ the formula (2.2) in both cases is only in qualitative agreement with the exact solution. It is shown in [16] that if the particle moves near the wall in a uniform shear flow (which to a certain extent models the flow in the boundary layer), then the linear term in (2.2) is preserved without changes. From the results presented in [9] it may be concluded that in this case the quadratic term also does not change. However, terms of higher order depend substantially on the nature of the flow of the carrying medium near the wall, so that in the analysis of the motion of a particle in the boundary layer on the sphere it is apparently meaningless to refine the formula (2.2) on the basis of model problems.

In this paper, in order to estimate the "effect of the wall" on the particle motion in the vicinity of the critical point, in the first and second equations of (1.3) C_D was replaced by C_{Dx} and C_{Dy}, respectively, for which the following relations were used:

$$C_{Dx} = C_D\left[1 + \frac{9}{16}\frac{r}{y} + \left(\frac{9}{16}\frac{r}{y}\right)^2\right], \quad C_{Dy} = C_D\left[1 + \frac{9}{8}\frac{r}{y} + \left(\frac{9}{8}\frac{r}{y}\right)^2\right], \tag{2.3}$$

where the value of C_D was calculated from the formulas (2.1). In addition, quite small particles, whose motion in the viscous boundary layer is close to a creep, were studied.

Figure 1 shows the trajectories of particles of two sizes in the vicinity of the critical point [1, 2) $r = 0.5\cdot10^{-4}$; 3, 4) $r = 0.54\cdot10^{-4}$]; $Re = 10^5$, $z_\infty = 10^{-5}$. The trajectories 1 and 3 correspond to the "standard curve" (2.1), trajectories 2 and 4 correspond to the formulas (2.3). We note that for the given ρ and Re the critical radius of the particles is $r_* \gtrsim 0.52\cdot10^{-4}$ [7]. The value of Re_p in the viscous boundary layer did not exceed 0.001 for trajectories 1 and 2 and 0.5 for trajectories 3 and 4. It is evident from Fig. 1, that if the size of the particle is even insignificantly greater than r_*, then the wall has practically no effect on its trajectory (the lines 3 and 4 coalesce) and, therefore, on the characteristics of inertial settling of this fraction. At the same time, if $r < r_*$, then this effect becomes appreciable. This especially affects the magnitude of the flux of the descending particles, which is determined by the coordinate x_w at the moment of settling. Thus in the case presented ($r = 0.5\cdot10^{-4}$) the difference in the flow is equal to $\sim$30%. In studying the settling of the particles, it evidently makes sense to study only those fractions whose flux is not vanishingly small, i.e., fractions with sizes $r > r_*$; at the same time, as follows from the results presented, the effect of the wall in the determination of the flux density of the particles at the critical point is insignificant. This result is also valid for other Reynolds numbers studied ($Re = 10^6$ and 10^7). The physical explanation for this is that in spite of the manyfold increase in C_D near the surface of the sphere, a particle with radius r larger than r_*, having a sufficient store of kinetic energy, "drifts" through the boundary layer within a very short time, so that the additional momentum due to the effect of the wall is much smaller than the intrinsic momentum of the particle. For $r < r_*$ this momentum is comparable to the momentum of the particle, and for this reason the change in the trajectory is appreciable.

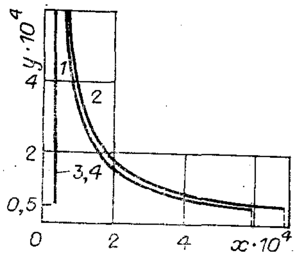

Fig. 1

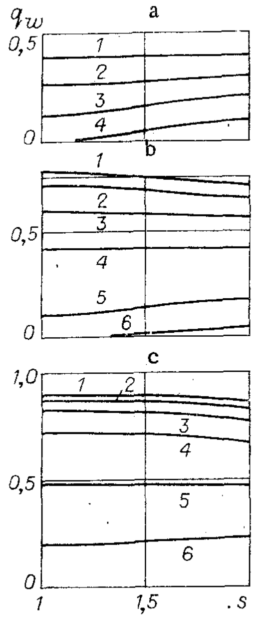

Fig. 2

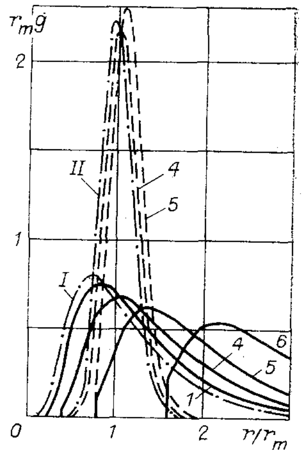

Fig. 4

Based on the estimate obtained above, the effect of the wall was ignored in the study of the inertial settling of polydispersed particles at the critical point and the "standard curve" (2.1) was used for C_D.

3. In the derivation of the formulas (3.4) and (3.6), all quantities, unless otherwise stated, are dimensional. Let us assume that in the unperturbed flow the average density of the dispersed phase $\rho_{p\infty}$ and its normalized distribution function over the fractions $g_\infty(r)$ are given. The density of the total particle flux in the unperturbed flow and the density of the flux of particles with radii varying from r to r + dr are evidently given by

$$q_\infty = \rho_{p\infty} V_\infty, \; dq_\infty(r) = q_\infty g_\infty(r) dr. \tag{3.1}$$

It is evident from the second relation that $g_\infty(r)$ is also a function of the flux density distribution q_∞. We denote by q_w and $dq_w(r)$ the total flux density of the dispersed particles and the flux density of particles with radii from r to r + dr, respectively, settling at the critical point. We introduced the function

$$q(r) = dq_w(r)/dq_\infty(r). \tag{3.2}$$

The quantity q(r) can be found if the trajectory of the particles of the given fraction is known. Indeed, let a particle with radius r, moving at a distance z_∞ from the axis of symmetry in the unperturbed flow (with $y = y_\infty$), settle onto the surface of a sphere at a point with the coordinate x_w. Then

$$q(r) = \lim_{z_\infty \to 0} (z_\infty / x_w)^2.$$

In the calculations the quantity q was calculated from the formula $q = (z_\infty/x_w)^2$, where the parameter z_∞ was fixed so that the dimensional value of x_w would not exceed 0.01. In this case, the accuracy of the indicated formula is very high.

From the relations (3.1) and (3.2) follows the equality

$$dq_w(r) = q_\infty q(r) g_\infty(r) dr. \tag{3.3}$$

Integrating (3.3) over r, we obtain

$$q_w = q_\infty \int_0^\infty q(r) g_\infty(r) \, dr, \tag{3.4}$$

where the limits 0 and ∞ conditionally denote the smallest and largest particle radii in the gas suspension.

We introduce the normalized flux density distribution function of particles settling at the critical point $g_w(r)$ with the help of the equality

$$dq_w(r) = q_w g_w(r)dr. \tag{3.5}$$

Substituting into (3.5) the expression for $dq_w(r)$ from (3.3) and transforming, we obtain

$$g_w(r) = q_\infty q(r) g_\infty(r)/q_w. \tag{3.6}$$

We now transform to dimensionless variables, scaling q_w to q_∞, the radius of the particles as before to a, and g_∞ and g_w to $1/a$. Then the relations (3.4) and (3.6) will assume the form

$$q_w = \int_0^\infty q(r) g_\infty(r)\, dr, \quad g_w(r) = \frac{q(r) g_\infty(r)}{q_w}. \tag{3.7}$$

The function q(r) in the relations (3.7) was determined for each Reynolds number from the results of calculations of the particle trajectories for more than 50 fractions. The step along r was chosen to be nonuniform, so as to approximate the function q(r) with high accuracy in a piecewise linear form in a wide range of variation of the relative radius of the particles (r varied form $0.4 \cdot 10^{-5}$, when for the values of Re studied $q \approx 0$, up to $0.2 \cdot 10^{-2}$, when $q \approx 1$).

In the numerical study of the characteristics of inertial settling of polydispersed particles, the function $g_\infty(r)$ was chosen in the form of the log-normal law [17], which in dimensionless variables has the form

$$g_\infty(r) = \frac{1}{\sqrt{2\pi}\, r \ln s} \exp\left[-\left(\frac{\ln r - \ln r_m}{\sqrt{2} \ln s}\right)^2\right], \tag{3.8}$$

where $\ln r_m$ and $\ln s$ are the mathematical expectation and the rms deviation of the logarithm of the relative radii of the particles in the mixture. The distribution function (3.8) is determined by two independent parameters r_m and s. If, however, r is replaced by the argument r/r_m, then the product $r_m g$ will be determined solely by the parameter s and for a fixed value of s the unperturbed distribution function in the coordinates $(r/r_m, r_m g)$ will therefore be independent of r_m. Such coordinates are convenient for comparing the flux density distribution functions of the settling particles, obtained for different values of r_m, with the unperturbed distribution law, and they shall be used below.

Figure 2 shows the dependence of the dimensionless density of the total flux of polydispersed particles settling at the critical point on the quantity s for different values of r_m (in a, b, and c Re = 10^5, 10^6, 10^7, respectively). Curves 1-6 correspond to $r_m \cdot 10^4$ = 1.0; 0.8; 0.6; 0.4; 0.2; 0.1. The quantity q_w at s = 1 is equal to $q(r_m)$, i.e., the density of the relative flux of monodispersed particles with radius r_m. As is evident from the results presented, variation of the magnitude of the spread in the particle sizes relative to the given value r_m over a wide range ($1 \leqslant s \leqslant 2$) has virtually no effect on the value of q_w, if $q_w \gtrsim 0.2$. The difference in q_w for mono- and polydispersed particles with a large size spread (s = 2) in this case is not more than 10%, and for some combinations of Re and R_m, q_w is virtually independent of s. Thus for Re = 10^5, $r_m = 1.0 \cdot 10^{-4}$ (curve 1 in Fig. 2a) and Re = 10^6, $r_m = 0.4 \cdot 10^{-4}$ (curve 4 in Fig. 2b) q_w deviates by about 1% when s varies from 1 to 2. We note that in both cases the total flux density is approximately the same ($q_w \approx 0.4$).

The distribution function of the particle flux density at the critical point over the size fractions with Re = 10^6 are shown in Fig. 3. Curves 1 and 4-6 refer to the same values of r_m as does Fig. 2. The solid lines correspond to s = 1.8 and the broken lines correspond to s = 1.2. The dot-dash lines show, for comparison, the distribution functions in the undispersed flow (I, II — s = 1.8; 1.2). From the curves presented it is evident that the graph of the function g_w for all r_m is shifted toward higher values of r than for g_∞. At the same time, the distribution law g_w approaches the unperturbed law as r_m increases. The noted shift in g_w is explained by the fact that the larger particles are not deflected as strongly by the gas flow and their relative fraction increases in the presence of settling. The largest deviation of the distribution function g_w from the unperturbed function is observed in the case when the size of the "representative" fraction in the starting mixture is too small, so that either they are strongly deflected by the carrying gas and fly past the bow surface without settling or their settling is insignificant. In this case, only quite large fractions, of which there are very few in the incident flow, settle. It is evident (see Fig. 2) that even the density of the total flux of settling particles is small in this case.

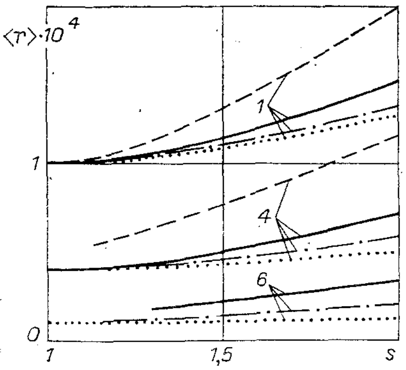

Fig. 4

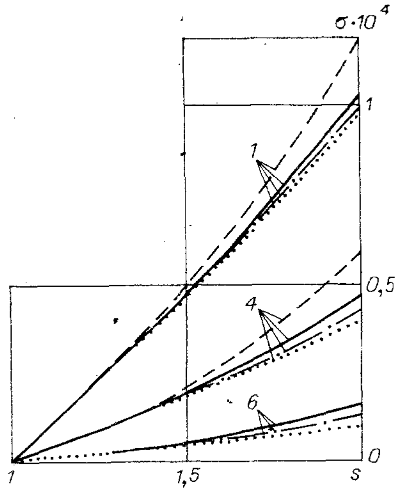

Fig. 5

For some value of r/r_m the function g_w changes in an almost jump-like fashion (see, for example, the curve 6 in Fig. 3 with $r/r_m = 1.6$). This effect is linked to the influence of the viscous boundary layer on the sphere, which gives rise to a sharp decrease in the particle flux toward the surface as the particle radius decreases, beginning with the critical value r_* [7] (in this case $r_* = 0.16 \cdot 10^{-4}$). If there are many particles with radii less than the critical value in the starting mixture, then the sharp drop in the function is distinct, as in the indicated example. As the number of small particles decreases, the picture becomes somewhat "smeared."

Calculations of the distribution functions g_w were also performed for other values of the parameters r_m, s, and R_e. More than 200 variants were calculated. The mathematical expectation $\langle r \rangle_w$ and the rms deviation σ_w of the radii of the polydispersed particles settling at the critical point were found for the distribution functions obtained. Some typical results are shown in Figs. 4 and 5. Curves 1, 4, and 6 refer to the same values of r_m as in Fig. 2. The dashed lines correspond to $Re = 10^5$, the solid lines correspond to $Re = 10^6$, and the dot–dash lines correspond to $Re = 10^7$. For comparison, the mathematical expectations $\langle r \rangle_\infty$ and the rms deviations σ_∞ of the particle radii, corresponding to the undisturbed distribution function (3.8), are also presented in the figures (dotted lines). The dependences of $\langle r \rangle_w$ and σ_w on s for $Re = 10^5$, $r_m = 0.4 \cdot 10^{-4}$ and $Re = 10^6$, $r_m = 0.1 \cdot 10^{-4}$ in the interval $1 \leqslant s \lesssim 1.2$ are not given because of the low accuracy of the results. The unsatisfactory accuracy of the determination of $\langle r \rangle_w$ and σ_w for the indicated values of Re, r_m, and s is linked to the high relative error in the calculation of q_w, which stands in the denominator in formula (3.7) for g_w, when this quantity is close to zero (see Fig. 2).

As is evident from the results presented, the rms deviation of the radii of the settling particles is for $1 \leqslant s \lesssim 1.5$ a more conservative characteristic than the average radius $\langle r \rangle_w$. Indeed, in the indicated range of variation of s for all Re numbers and values of r_m studied, we have $\sigma_w \gtrsim \sigma_\infty$, while the quantity $\langle r \rangle_w$ depends quite markedly on Re and can differ substantially from $\langle r \rangle_\infty$ (for example, for $Re = 10^6$, $r_m = 0.1 \cdot 10^{-4}$ and $s = 1.5$ the average particle radii in the unperturbed flow and at the critical point differ by a factor of 2). The difference between σ_w and σ_∞ increases with s.

For fixed r_m and s, Re approach $\langle r \rangle_w$ and σ_w with the corresponding unperturbed values as Re increases. The small deviation of $\langle r \rangle_w$ and σ_w from $\langle r \rangle_\infty$ and σ_∞ indicates that for the given value of Re almost all particles in the gas suspension are weakly deflected by the gas flowing past the sphere and their trajectories are nearly rectilinear.

In conclusion, it should be noted that the proposed method for determining the flux density of inertially settling polydispersed particles and their size distribution function, based on the calculation of the trajectories of particles of separate sizes, is easily generalized to other points of the bow surface.

LITERATURE CITED

1. N. A. Fuks, Mechanics of Aerosols [in Russian], Izd. Akad. Nauk SSSR, Moscow (1955).
2. H. Herne, "The classical computations of the aerodynamic capture of particles by spheres," in: Aerodynamic Capture of Particles, Proc. Conf. held at BCURA, Leatherhead, Surrey, 1960, Pergamon Press, Oxford (1960).

3. L. M. Levin, Investigations in the Physics of Coarsely Dispersed Aerosols [in Russian], Izd. Akad. Nauk SSSR, Moscow (1961).
4. V. M. Voloshchuk, Introduction to the Hydrodynamics of Coarsely Dispersed Aerosols [in Russian], Gidrometeoizdat, Leningrad (1971).
5. S. A. Morsi and A. J. Alexander, "An investigation of particle trajectories in two-phase flow systems," J. Fluid Mech., 55, Pt. 2 (1972).
6. A. M. Mkhitaryan, Yu. V. Sukharnikov, and Yu. G. Koval', "Study of the settling of drops on cylindrical bodies moving in an aerosol cloud," in: Aerodynamics [in Russian], No. 3, Kiev (1977).
7. Yu. M. Tsirkunov, "Effect of a viscous boundary layer on the settling of particles in the flow of a gas suspension past a sphere," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1 (1982).
8. Yu. M. Tsirkunov, "Approximation of the parameter fields of a viscous carrying medium in the study of the flow of a dispersed mixture with large Reynolds numbers past a blunt body," in: Dynamics of Inhomogeneous and Compressible Media [in Russian], Izd. Leningr. Gos. Univ., Leningrad (1984).
9. J. Happel and H. Brenner, Hydrodynamics at Low Reynolds Numbers [Russian translation], Mir, Moscow (1976).
10. R. I. Nigmatulin, Foundations of the Mechanics of Heterogeneous Media [in Russian], Nauka, Moscow (1978).
11. M. Van Dyke, "High approximations in boundary-layer theory. Part 1. General analysis," J. Fluid Mech., 14, Pt. 2 (1962).
12. N. E. Kochin, I. A. Kibel', and N. V. Roze, Theoretical Hydromechanics [in Russian], Fizmatgiz, Moscow (1963), Pt. 1.
13. G. Shlikhting, Boundary Layer Theory [in Russian], Nauka, Moscow (1974).
14. R. V. Hemming, Numerical Methods [Russian translation], Nauka, Moscow (1972).
15. S. H. Lee, R. S. Chadwick, and L. G. Leal, "Motion of a sphere in the presence of a plane interface. Part 1. An approximate solution by generalization of the method of Lorentz," J. Fluid Mech., 93, Pt. 4 (1979).
16. A. J. Goldman, R. G. Cox, and H. Brenner, "Slow viscous motion of a sphere parallel to a plane wall. II. Couette flow," Chem. Eng. Sci., 22, No. 4 (1967).
17. P. A. Kouzov, Foundations of the Analysis of the Dispersed Composition of Industrial Dusts and Pulverized Materials [in Russian], Khimiya, Leningrad (1971).

SUPERSONIC FLOW OF A GAS SUSPENSION NEAR A WEDGE IN THE PRESENCE OF REFLECTED PARTICLES

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Supersonic flow, perturbed by the interaction of the gas with a cloud of monodispersed particles, near a wedge is studied. The exact solution of the problem of the motion of particles behind an oblique shock and particles specularly reflected from the surface of the wedge is given. These results are used to determine the perturbations of the gasdynamic parameters and forces acting on the wedge in a two-phase flow. The effects of the particles on the flow in two different situations are compared: In one situation the particles stick to the surface of the wedge; in the other situations they reflect elastically, and form a layer of dust with a sharp contact boundary.

The problem of the perturbed gas flow behind an oblique shock was previously studied in gasdynamics [1] and in the dynamics of a radiating gas [2]. The problem of supersonic two-phase flow near a wedge was studied on the basis of the linear theory in [3] and by numerical methods in [4]. The solution of the problem of the motion of a particle behind an oblique shock [5] and of a reflected particle [6] are known. The exact solution of the problem of the motion of a cloud of particles behind an oblique shock was found in [7].

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